On the DM annual modulation signal

(in collaboration with N. Bozorgnia, T. Schwetz and J. Zupan)

[JCAP 03 (2012) 005, 1112.1627; PRL 109 (2012) 141301, 1205.0134; 0717681]

Juan Herrero-García

IFIC, Universidad de Valencia - CSIC

(visiting Northwestern University)

Fermilab

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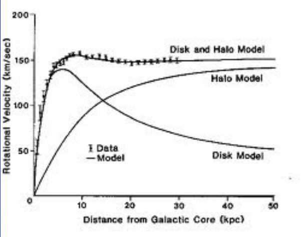
Outline

1. Evidence and properties of dark matter
2. Annual modulation in direct searches
3. Bounds on the annual modulation and results
4. Bounds between different experiments and results
5. Inelastic scattering results
6. Final remarks and conclusions

EVIDENCE AND PROPERTIES OF DARK MATTER

Evidence for dark matter

- Rotation curves of spiral galaxies (Milky Way - 21cm line):

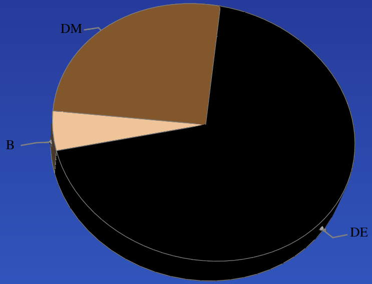


- Bullet cluster (X-rays + gravitational lensing):

CMB data

Planck's results alone:

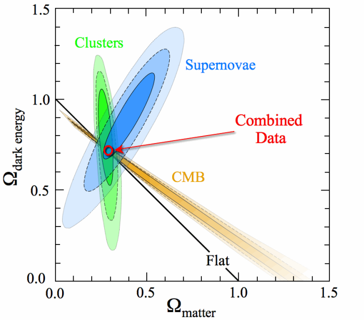
- $\Omega_{DE} \approx 0.69$
- $\Omega_B \approx 0.05$
- $\Omega_{DM} \approx 0.26$ (anisotropies of CMB)



The colours:

reflect how clear our current knowledge of the different components is.

CMB data



Concordance Model:

- CMB $\rightarrow \Omega_{TOT} = 1$
- SNIA $\rightarrow \Omega_{DE}, \Omega_M$
- BBN $\rightarrow \Omega_B$
- Clusters $\rightarrow \Omega_M$

$\longrightarrow \Omega_{DM}$

Other evidence:

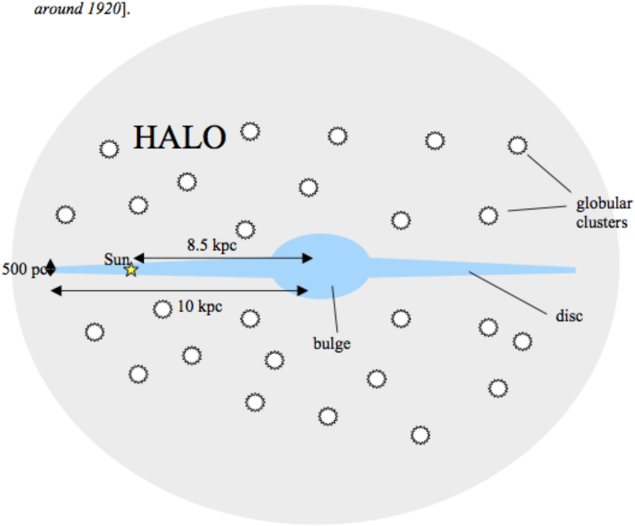
- M/L ratio in galaxy clusters (virial theorem to gas).
- Growth of structure (N-body simulations).
- Globular clusters in galaxies.

Properties of a DM particle (or particles)

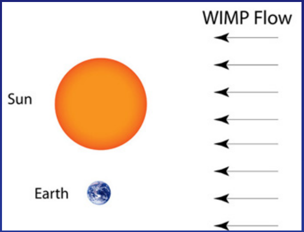
1. Interacts gravitationally.
2. With current's observed abundance (long-lived/ stable).
3. Neutral (no e.m. interactions at tree level).
4. If it acts weakly with the SM, direct detection is possible.
5. Cold (or warm), otherwise would have free-streamed erasing small scales.
6. Collisionless: it does not dissipate, it forms haloes.

The Milky Way

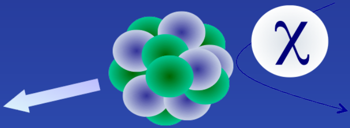
Using Cepheid variables and other techniques (e.g. a type of main-sequence fitting) the structure and scale of the Milky Way was first determined [*by Harlow Shapley around 1920*].



ANNUAL MODULATION IN DIRECT SEARCHES



As we move through the galaxy, in our detector rest frame there is a WIMP wind.

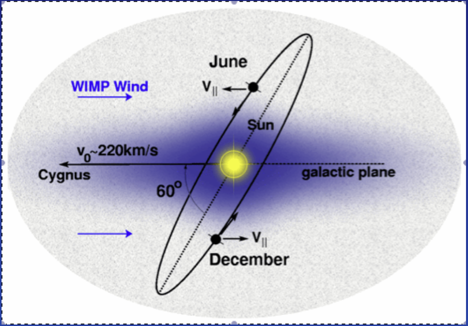


If DM interacts weakly, it can produce nuclear recoils.

Extremely difficult experiments:

- Underground to reduce background.
- Energy deposited via ionization, heat &/or light.

Annual modulation in direct searches



Annual modulation:

Depending on the time of the year, we should *receive* more or less DM flux scattering in our detectors.

Typical velocities involved:

- $v_{esc} \simeq 550\,\text{km/s}$.
- $\bar{v} \simeq v_{Sun} \approx 220\,\text{km/s}$.
- $v_e(t) \propto v_e \cos 2\pi(t - t_0)$, with $v_e \simeq 30\,\text{km/s}$.

Direct detection event rate: notation

- Flux (♯ particles/ area/ time), with $\rho \approx 0.3\,\mathrm{GeV/cm^3}$:

$$\phi_\chi = n_\chi v = \frac{\rho_\chi}{m_\chi} v = \left(\frac{100\,\mathrm{GeV}}{m_\chi}\right)\, 10^5 \mathrm{cm^{-2} s^{-1}}$$

- Hand - waving rate (♯ counts/ time):

$$R = \phi_\chi\, \sigma_\chi\, N_{target} = \frac{\rho_\chi v}{m_\chi}\, \sigma_\chi\, \frac{\text{target mass}}{m_A}$$

- Differential event rate (♯ counts/ keV/ kg/ day):

$$R(E_r, t) = \frac{\rho_\chi}{m_\chi m_A} \int_{v_m} d^3v\, \frac{d\sigma_\chi}{dE_r}\, v\, f_{det}(\vec{v}, t)$$

where by kinematics ($\delta = m_{\chi *} - m_\chi$; for elastic $\delta = 0$):

$$v_m = \sqrt{\frac{m_A E_r}{2\mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2 m_A E_r}}$$

Event rate final: simple expression

- The velocity distribution fulfills ($\int d^3v\, f_{det}(\vec{v}, t) = 1$):

$$f_{det}\,(\vec{v}, t) = f_{Sun}(\vec{v} + \vec{v_e}(t)) = f_{gal}(\vec{v} + \vec{v}_S + \vec{v_e}(t)) \geq 0.$$

- Using that for spin-independent (SI):

$$\frac{d\sigma_\chi}{dE_r} = \frac{m_A}{2\mu_{\chi A}^2 v^2} F^2(E_r)\sigma_A^0,$$

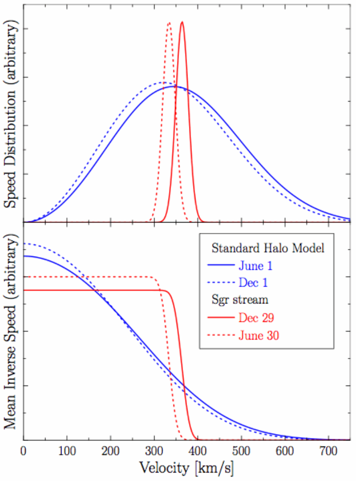
where $\sigma_A^0 = \sigma_p[Z + (A - Z)(f_n/f_p)]^2 \mu_{\chi A}^2/\mu_{\chi p}^2$.

- The final rate can be simplified as:

$$R(E_r, t) \equiv C\, F^2(E_r)\, \eta(v_m, t), \quad \text{with} \quad C \equiv \rho_\chi \sigma_A^0 / 2 m_\chi \mu_{\chi A}^2,$$

and:

$$\eta(v_m, t) \equiv \int_{v_m} d^3v\, \frac{f_{det}(\vec{v}, t)}{v}.$$



N-body simulations, at low velocities, DM in equilibrium, smooth halo:

- SHM - isothermal sphere with isotropic, Maxwellian $f(\vec{v})$:

$$f^{gal}_{SHM}(\vec{v}) \propto e^{-\vec{v}^2/\bar{v}^2}$$

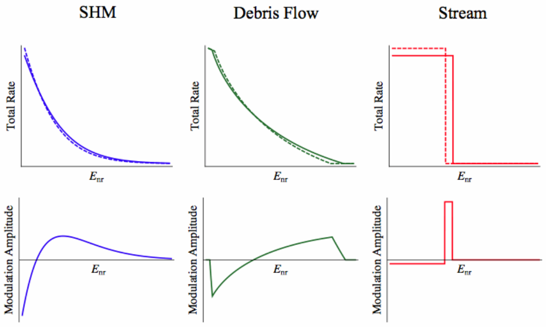
- Therefore, spectrum is exponential in the lab. frame:

$$R \sim e^{-E_r/E_0}$$

with $E_0 \sim \mathcal{O}(10\,\mathrm{KeV})$.

Typical rates and annual modulations

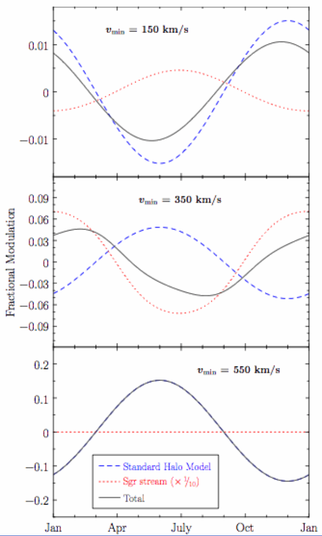
$$A_R(E_r) \approx 0.5\,[R(E_r, \text{June}) - R(E_r, \text{December})]$$



There can be unvirialized substructure at high velocities, important for low mass DM or inelastic scattering:

- Streams - small v_{disp}, not spatially mixed: $\propto \delta^3(\vec{v} - \vec{v}_{stream})$
- Debris flows - spatially homog. velocity: $\propto \delta(|\vec{v}| - v_{flow})$

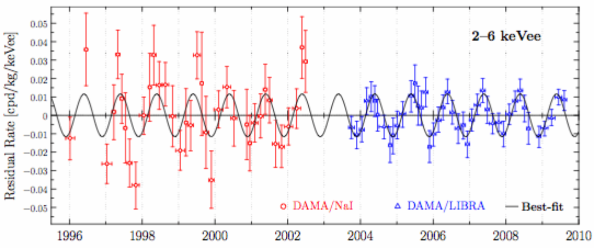
Modulation fraction = mod. / rate: $S_m = A_R / R$



Features:

- SHM: $\mathcal{O}(10\%)$, sinusoidal, $t_0 \sim$ June, flip $\sim v_{min} \approx 200$ km/s.
- E_R of flip constrains m_χ.
- At large v_m, S_m grows, but no sensitivity except for very low m_χ.
- Streams: A_R large at $v_m \approx v_{stream}$. Possibly non-sinusoidal (higher harmonics). t_0 varies.
- Presumably $f(v)$ is a mixture
- Need to be independent of $f(v)$.

DAMA's and CoGeNT's annual modulation



CoGeNT (Ge):

- $2.8\,\sigma$
- phase April 16.
- $S_m \sim 0.1 - 0.3$.

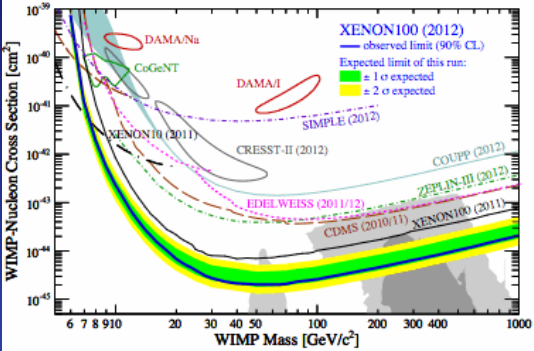
$\longrightarrow m_\chi \sim 10$ GeV.

DAMA (NaI):

- $8.9\,\sigma$, consistent with SHM phase at June 1.
- Fits typical sinusoidal modulation with $T = 1$ year.
- $S_m \sim 0.02$

$\longrightarrow m_\chi \sim 10$ GeV (Na) & $m_\chi \sim 80$ GeV (I)

DAMA & CoGeNT vs other experiments (Aprile *et al.*)



It assumes a SHM, a local density and escape velocity:

- XENON: most stringent constraint on σ_{SI} for $m_\chi > 10$ GeV.
- $m_\chi \sim 80$ GeV (I) DAMA solution seems to be ruled-out for SI and SD by XENON, CDMS, COUPP.
- Need to be astrophysics independent!

BOUNDS ON THE ANNUAL MODULATION AND RESULTS

[JCAP 03 (2012) 005, 1112.1627]

Our goal: is the annual modulation seen due to DM?

1. First part: establish a consistency check between the *modulated* signal and the *constant* rate, that must be fulfilled *within an experiment* by dark matter, by making very mild assumptions about the DM halo. [JCAP03(2012)005, 1112.1627 [hep-ph]]
2. Second part: translate the bound on the rate of one experiment into a bound on the annual modulation in a *different experiment*. [PRL, 1205.0134 [hep-ph]]
3. Third part: *inelastic* scattering analysis of compatibility of DAMA and XENON100. [0717681 [hep-ph]]

Idea

- The cause of the modulation, i.e., the velocity of the Earth (assumed to have all the time dependence) wrt the Sun:

$$v_e \sim 30 \text{ km/s}$$

is much smaller than the typical DM velocity wrt Earth:

$$\langle v \rangle \sim 200 \text{ km/s}$$

- For typical $E_R \sim 10$ KeV and Na, I, Ge:

$$v_{esc} > \langle v \rangle > v_m \gg v_e$$

so we can expand $\eta(v_m, t)$ in $\dfrac{v_e}{v} \ll 1$.

Expansion of $\eta(v_m, t)$ in v_e/v

To first order, $\mathcal{O}(v_e/v)$:

$$\eta(v_m, t) = \int_{v_m} d^3v\, \frac{f_{det}(\vec{v})}{v} = \int_{v_m} d^3v\, \frac{f_{Sun}(\vec{v} + \vec{v_e}(t))}{v} =$$

$$= \int_{v_m} d^3v\, \frac{f_{Sun}(\vec{v})}{v} +$$

$$+ \int d^3v\, f_{Sun}(\vec{v})\, \frac{\vec{v}\cdot\vec{v_e}(t)}{v^3} [\Theta(v - v_m) - \delta(v - v_m)\, v_m] \equiv$$

$$\equiv \bar{\eta}(v_m) + A_\eta(v_m) \cos 2\pi(t - t_0)$$

- $\bar{\eta}(v_m)$ is constant, A_η is modulated, with observed rates:
  $\overline{R} \equiv CF^2(E_r)\bar{\eta}(v_m)$ and $A_R \equiv CF^2(E_r)A_\eta$
- Can check for convergence with higher orders (Bozorgnia, H-G, Schwetz, Zupan, work in progress).

The general bound on the annual modulation

Assumptions:

1. Halo "smooth" on scales $\lesssim v_e \sim 30\,\text{km/s}$: streams with $v_{disp} < v_e$ not covered.
2. Only time dependence in $v_e(t)$, not in f_{Sun} (no change on months), so spatially constant ρ at Sun-Earth scale.

$$A_\eta(v_m) \leqslant v_e \left[-\frac{d\bar{\eta}}{dv_m} + \frac{\bar{\eta}(v_m)}{v_m} - \int_{v_m} dv \frac{\bar{\eta}(v)}{v^2} \right]$$

Integrating over v_{min} and dropping the negative term:

$$\int_{v_{m1}}^{v_{m2}} dv_m \, A_\eta(v_m) \leqslant v_e \left[\bar{\eta}(v_{m1}) + \int_{v_{m1}} dv \, \frac{\bar{\eta}(v)}{v} \right]$$

- r.h.s. in terms of observed rates. Irrespective of phase.
- Allows an arbitrary halo structure, including several streams from different directions.

Symmetric bounds

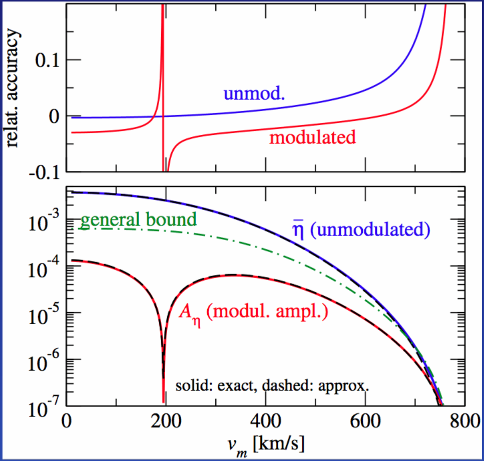
- Preferred constant direction $\hat{v}_{HALO}$ independent of v_m of the DM velocity distribution in the Sun's rest frame:

$$\int_{v_{m1}}^{v_{m2}} dv_m \, A_\eta(v_m) \leqslant v_e \left[\bar{\eta}(v_{m1}) - v_{m1} \int_{v_{m1}} dv \, \frac{\bar{\eta}(v)}{v^2} \right]$$

- Fulfilled for isotropic halos (Maxwellian), streams parallel to the motion of the Sun like a dark disc... Phase constant (up to sign flip).
- For these popular cases $\hat{v}_{HALO} \propto \hat{v}_{SUN}$, with $t_0 =$ June 1st:

$$\int_{v_{m1}}^{v_{m2}} dv_m \, A_\eta(v_m) \leqslant 0.5 \, v_e \left[\bar{\eta}(v_{m1}) - v_{m1} \int_{v_{m1}} dv \, \frac{\bar{\eta}(v)}{v^2} \right]$$

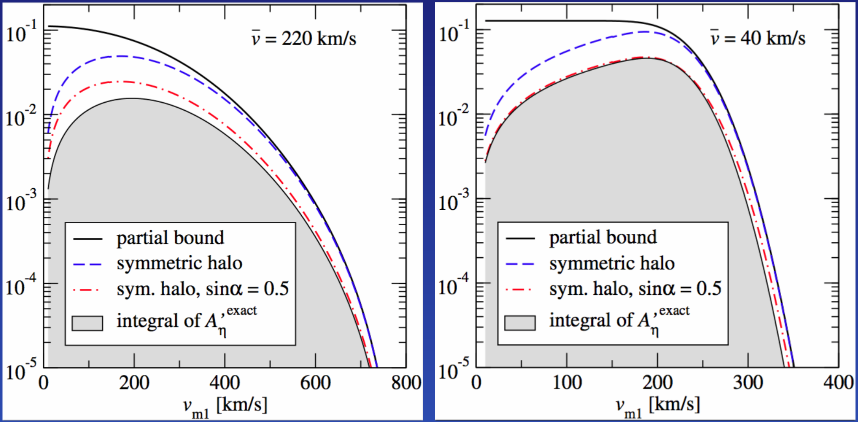
Checking the general bound for the Maxwellian halo



Maxwellian halo is far from saturating the general bound.

Bound one order of magnitude more stringent than $A_R < \bar{\eta}$.

Checking the symmetric bounds



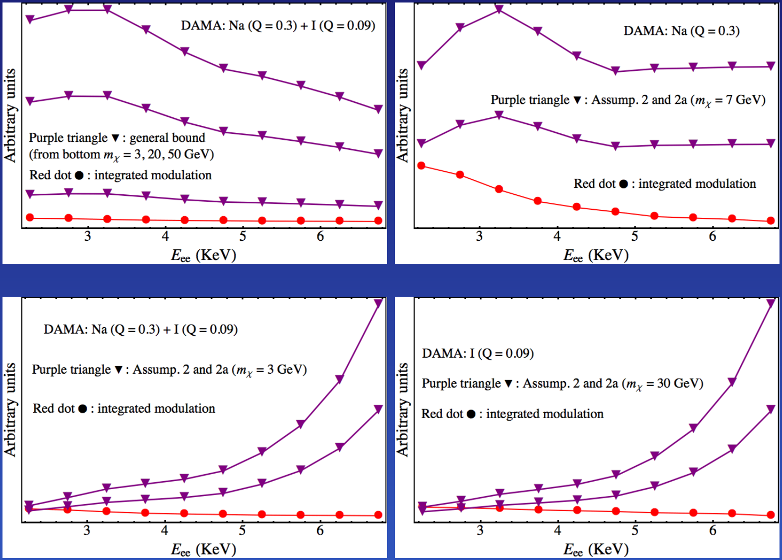
Symmetric bounds are even stronger.

Close to dispersion velocities $\sim v_e$, the bounds are saturated.

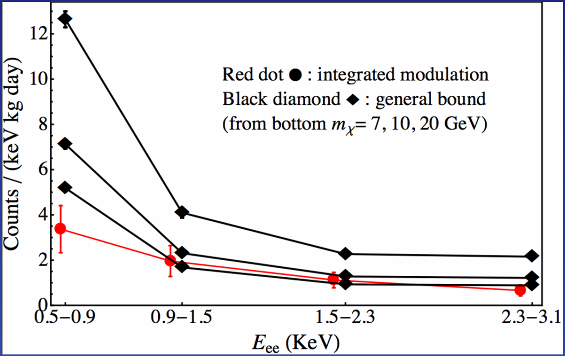
Subtleties:

- Experimental data binned: average over each bin.
- Continuum bounds vs discrete data: integrals $\rightarrow$ sums...
- Single-target detector (CoGeNT) or multi-target (DAMA).
- Dependence on ρ_χ, σ_p, v_{esc} drops from the bounds.
- Depend on m_χ, $q(E_r)$ and $F^2(E_r)$.
- Valid for SI, SD and IV.

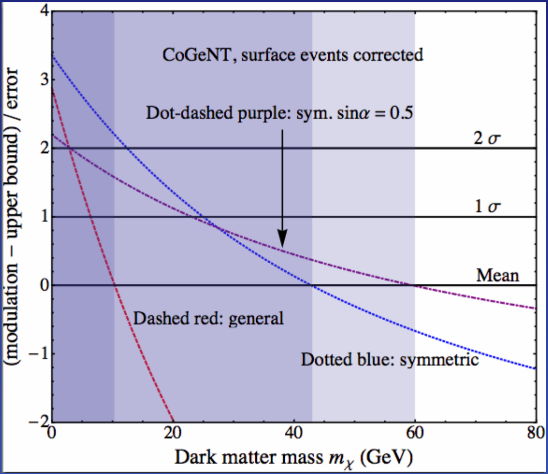
Results for DAMA: consistent with its rate



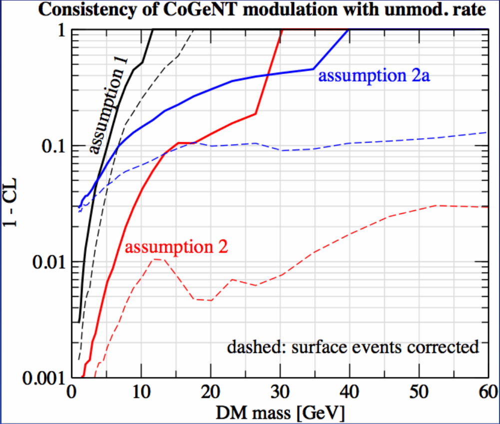
Results for CoGeNT: tensions with its rate



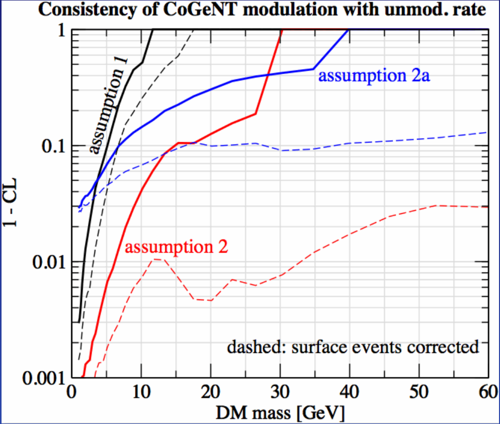
CoGeNT (with surface events subtracted)



Probability that the bound is fulfilled



Probability that the bound is fulfilled



CoGeNT is under severe pressure:

Under ass. 2 (2a) (typical DM haloes) data is inconsistent with any m_χ at $\gtrsim$ 97% C.L. ($\gtrsim$ 90% C.L.) respectively.

BOUNDS BETWEEN DIFFERENT EXPERIMENTS AND RESULTS

[PRL 109 (2012) 141301, 1205.0134]

The bounds are detector independent! (P. J. Fox *et al.*)

- Bounds are in terms of $\bar{\eta}$ or A_η, so are detector-independent, they can be applied to different exp.:

$$\int_{v_{m1}}^{v_{m2}} dv_m\, \tilde{A}_\eta^{DAMA}(v_m) \leqslant v_e\, \tilde{\eta}^{XENON}(v_{m1})$$

where we defined, with $\tilde{C} \equiv \rho_\chi \sigma_p / 2 m_\chi \mu_{\chi p}^2$:

$$\tilde{\eta}(v_m) \equiv \tilde{C}\, \bar{\eta}(v_m), \quad \& \quad \tilde{A}_\eta(v_m) \equiv \tilde{C}\, A_\eta(v_m)$$

- Assuming scattering on Na (low m_χ):

$$A_R^i = \frac{A_{Na}^2 \left\langle F_{Na}^2 \right\rangle_i f_{Na}}{q_{Na}} \tilde{A}_\eta^i(v_m^i)$$

where $q_{Na} = 0.3$ is the quenching factor, $F_{Na}(E_r)$ the form factor and $f_{Na} = m_{Na}/(m_{Na} + m_I)$ the Na mass fraction.

Upper bounds on $\tilde{\eta}(v_m)$ for null-result experiments

- The predicted number of events in $[E_1, E_2]$ is:

$$N^{pred}_{[E_1,E_2]} = MTA^2 \int_0^\infty dE_{nr} F_A^2(E_{nr}) G_{[E_1,E_2]}(E_{nr}) \tilde{\eta}(v_m)$$

with G the detector response, M the mass and T the exp. time.

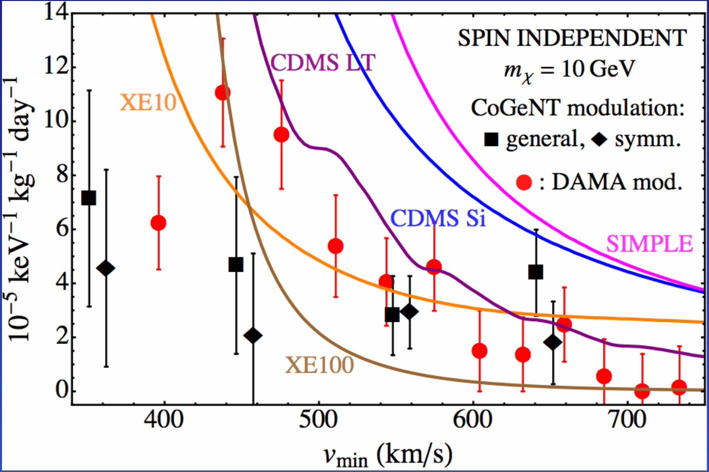
- As $\tilde{\eta}(v_m)$ is a falling function, minimum events when $\tilde{\eta}(v) \equiv \tilde{\eta}(v_m)\Theta(v_m - v)$ [P. J. Fox *et al.*].
- So, at v_m, there is a lower bound $N^{pred}_{[E_1,E_2]} \geq \mu(v_m)$, with

$$\mu(v_m) = MTA^2 \tilde{\eta}(v_m) \int_0^{E(v_m)} dE_{nr} F_A^2(E_{nr}) G_{[E_1,E_2]}(E_{nr})$$

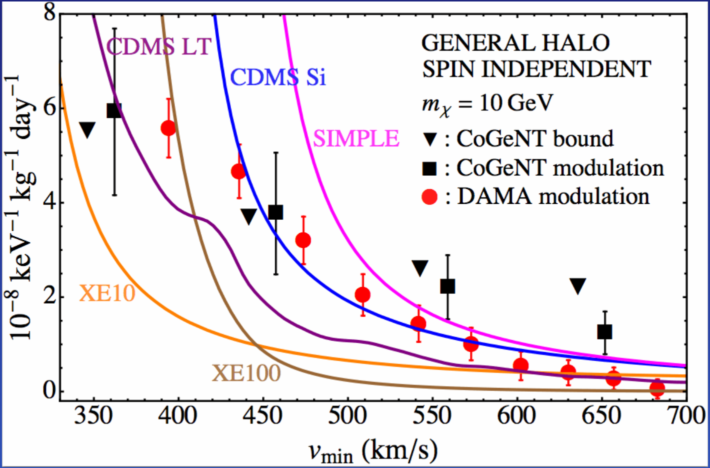
Can obtain an upper bound of at a given C.L. by:

requiring that the probability of obtaining $N^{obs}_{[E_1,E_2]}$ events or less for a Poisson mean of $\mu(v_m)$ is equal to 1-C.L.

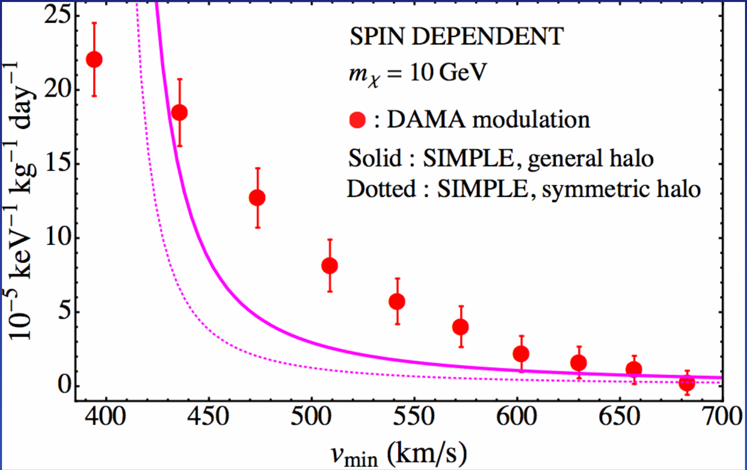
Modulations and upper bounds on the rates



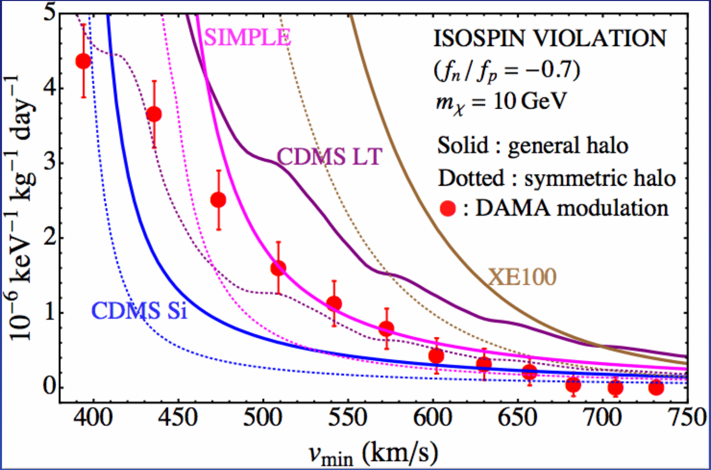
General bound, spin independent



Spin dependent (on protons)



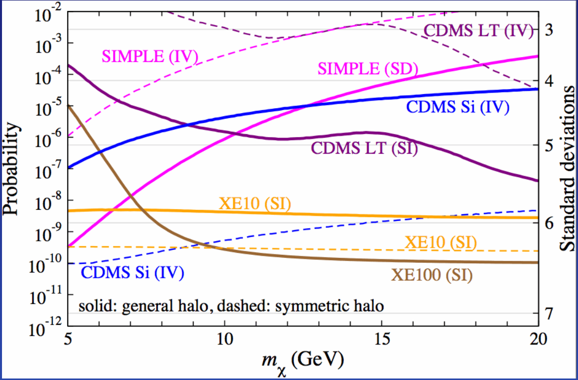
Isospin violation: different couplings to p and n.



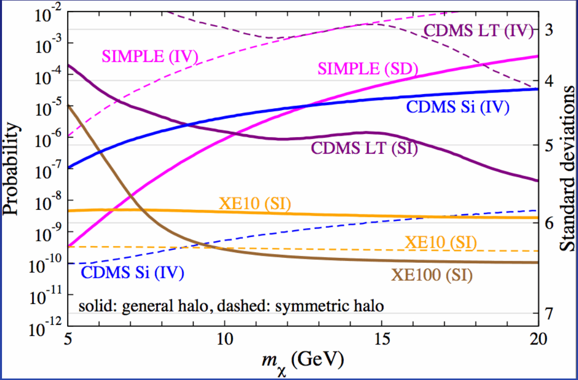
IV also excluded:

Can supress scattering on Xe, but then on Si it is large.

Probability of compatibility of DAMA's modulation with the other null-result experiments



Probability of compatibility of DAMA's modulation with the other null-result experiments



DAMA is inconsistent with other null-result experiments:

- $m_\chi \lesssim 15$ GeV, is disfavoured by ≥ 1 experiment at $\geq 4\sigma$.
- XE100 excludes at $> 6\sigma$ for $m_\chi \gtrsim 8$ GeV (SI).

Caveat: systematic uncertainties

We got rid (almost) of astrophysical uncertainties, however the bounds are still subjected to:

- particle physics: interaction type...
- nuclear: q_{Na}, F...
- experimental uncertainties: L_{eff}, channeling...

For example, DAMA's quenching factor...

For $q_{Na} = 0.45$:
- SI: excluded at $> 5\sigma$ for $m_\chi \gtrsim 10$ GeV (general halo).
- SD a IV can achieve a consistency at $\approx 3\sigma$ (general halo).

INELASTIC SCATTERING

[hep-ph: 0717681]

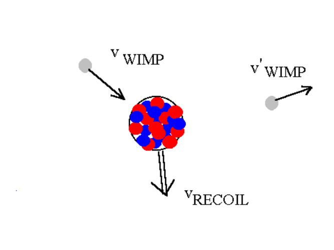
Concept of inelastic scattering

- A DM particle χ scatters to an excited state $\chi*$, with a mass difference:

$$\delta = m_{\chi^*} - m_\chi \sim \mathcal{O}(100\,\mathrm{KeV})$$

- Heavy nucleus are favoured: I in DAMA.
- $v_m \sim v_{esc}$, so only DM in the tail of DM distribution is probed, with:

$$v \in [v_{esc} - \Delta v,\ v_{esc}] \text{ with } \Delta v \sim v_e$$

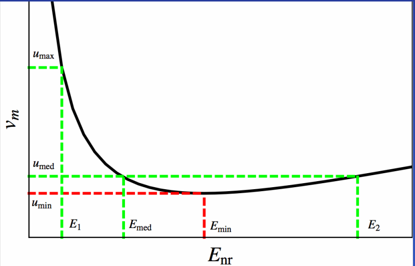


Non-unique $v_m \to E_R$ relation: shape test

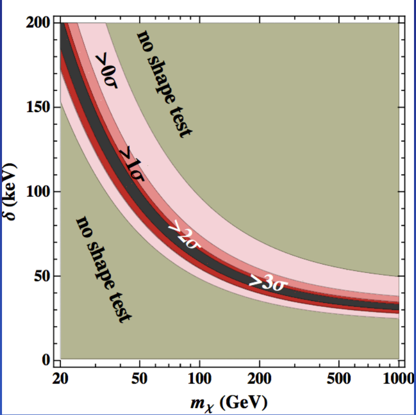
$$v_m = \sqrt{\frac{m_A E_r}{2\mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2 m_A E_r}}$$

$\int_{u_{min}}^{u_{med}} \to$ ambiguity:

$I_a = \int_{E_{min}}^{E_{med}}$ & $I_b = \int_{E_{min}}^{E_2}$ should be =!



$$\frac{|I_a - I_b|}{\sqrt{\sigma_a^2 + \sigma_b^2}}$$ for DAMA:

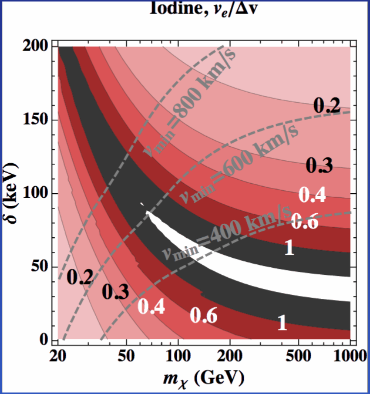


Range of validity of the bounds

- Bounds go like:

$$\langle A_\eta \rangle \lesssim \frac{v_e}{\Delta v} \langle \eta \rangle$$

so the expansion parameter $\sim v_e/\Delta v$ can become $\mathcal{O}(1)$.



- Δv - overlap between:

- XENON100 [6.61, 43.04] KeV.
- DAMA [2, 4] KeVee.

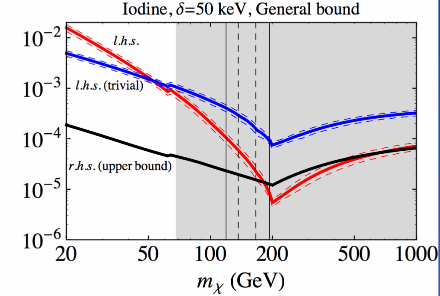
- Elastic ($\delta = 0$): $m_\chi \lesssim 50$ GeV.
- In all regions, the trivial bound applies, $A_\eta < \bar{\eta}$.

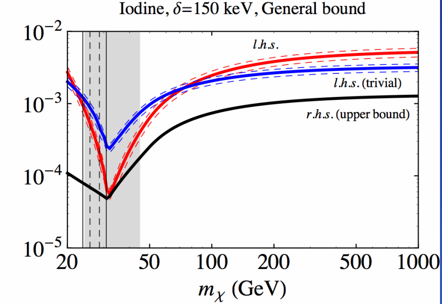
Inelastic results for the general bound for DAMA

- Integrated general bound:

$$\int_{u_{min}}^{u_{max}} dv\, A_\eta(v)(v - u_{min}) < 0.5\, v_e \left(3 - \frac{u_{min}^2}{u_{max}^2}\right) \int_{u_{min}}^{u_{max}} dv\, \bar{\eta}(v)$$

- Trivial bound: $\int_{u_{min}}^{u_{max}} dv\, A_\eta(v) < \int_{u_{min}}^{u_{max}} dv\, \bar{\eta}(v)$.





FINAL REMARKS AND CONCLUSIONS

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- 1) We have derived bounds (almost completely) astrophysics-independent between the annual modulation and the constant rate.

$\longrightarrow$ DAMA's modulation is consistent with its own rate, while CoGeNTs was incompatible with its own rate at $\gtrsim 90\,\%$ C.L.

- 2) We have extended the bounds to the case of comparing between the modulation in one experiment and the null-result of a different experiment.

$\longrightarrow$ DAMA, for all elastic interactions and with a DM mass $m_\chi \lesssim 15$ GeV, is disfavoured by ≥ 1 experiment at $\geq 4\sigma$.
$\longrightarrow$ Inelastic scatt. for DAMA strongly disfavoured by XE100.

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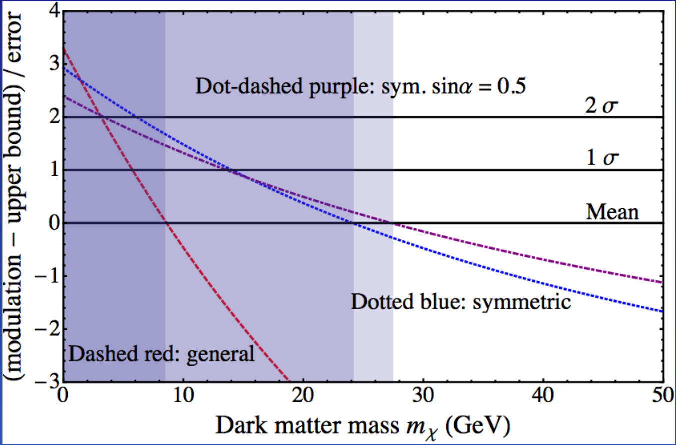
The bounds are a necessary (not sufficient) test:

Any annually modulated signal has to pass them before stating that it is due to dark matter.

THANKS!

BACK-UP

CoGENT (without subtraction of surface events)



To study the consistency between A and R

- Conservative approach: only a fraction ω_i ($0 \leq \omega_i \leq 1$) of R_i is due to DM, the rest being an unknown background.
- Build a "χ^2-like" function:

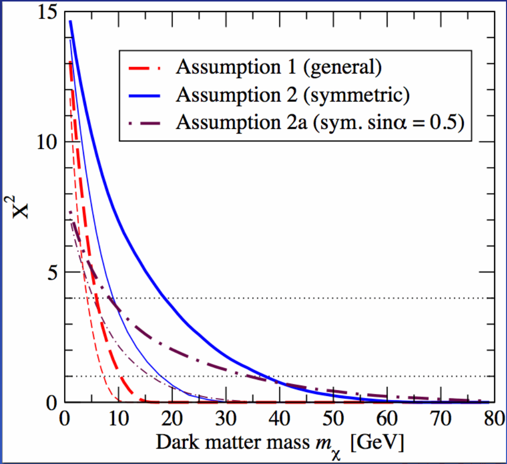
$$\Delta X^2 = \sum_i^N \left(\frac{A_i - B_i}{\sigma_i^A} \right)^2 \Theta(A_i - B_i)$$

and minimize w.r.t the ω_i.

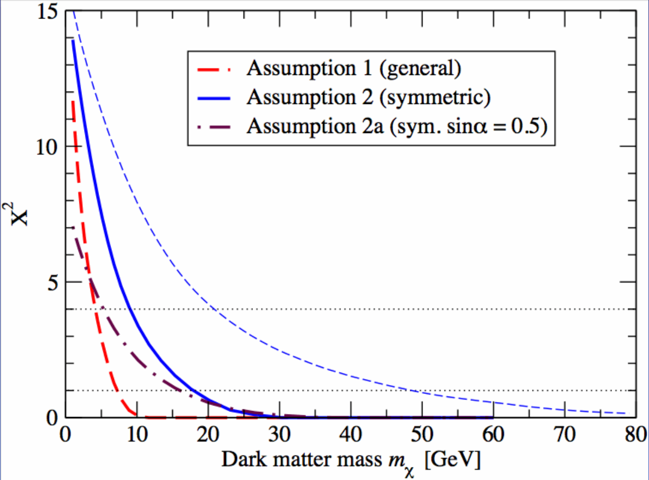
- There is only a contribution to it when the bound is violated.
- Approximately χ^2 distributed with 1 d.o.f., m_χ.

X^2 for CoGeNT (with & without surface events subtr.)

$$\Delta X^2 = \sum_i^N \left(\frac{A_i - B_i}{\sigma_i^A} \right)^2 \Theta(A_i - B_i)$$



Chi square minimization



CoGeNT bounds on the DM mass

	Proc. 1	Proc. 1	Proc. 2	Proc. 2
Mean mass (GeV)	Normal	Surface	Normal	Surface
General bound	8.5	10	7.3	10
Symmetric bound	24	43	18	37
Sym. $\alpha = \pi/6$	27.5	59.5	16	35

- ***Method 3:***

1. For each m_χ, compute the less constraining set of ω_i by minimizing the X^2.
2. With this set of ω_i, suppose the bound is saturated (conservative) and simulate pseudo-data (for the modulation) taking the upper bounds (r.h.s.) as the mean value for a Gaussian, with σ_i = error of the true A_i.
3. For each random data set, calculate the X^2 value and obtain its distribution.
4. Compare it with the X^2_{obs} of the real data and calculate the probability of obtaining a $X^2 > X^2_{obs}$.

- Probability to obtain $X^2 > X^2_{obs} \equiv P_{\text{bound is fulfilled}}$.

Iterative method

- ***Method 4.*** For each bin i, the inequality depends only on ω_j, with $j \geq i$. The most conservative option is to have ω_i (ω_j with $j > i$) as large (small) as possible.

Iterative prescription to find the set of ω_i corresponding to the most conservative choice of background:

1. Saturate the bounds ($\leq \rightarrow =$). System of N ($\sharp$ bins) linear equations in ω_i.
2. Starting with the highest bin $j = N$, solve for the ω_N that saturates the bound. If $\omega_N \leq 1$, it will be the smallest allowed value, so the bound for $N - 1$ will be the weakest. If $\omega_N \geq 1$, i it is violated & we set it to one.
3. Then go to the bin $j = N - 1$ with that value of ω_N and look for the ω_{N-1} that saturates the bound, and so on...

Iterative method bounds

	Proc. 4	Proc. 4
Mean mass (GeV)	Normal	Surface
General bound	10	12.5
Symmetric bound	29.5	63
Sym. $\alpha = \pi/6$	37.5	94.5

Quantifying DAMA's modulation discrepancy

- We fix v_m (or m_χ). For each $\tilde{\eta}(v_m)$ there is a Poisson mean $\mu(v_m)$. We calculate the probability p_η to obtain equal or less events than measured by the null-result experiment.
- We construct the bound (r.h.s.) using the same $\tilde{\eta}(v_m)$.
- We calculate the probability p_A that the bound is not violated by assuming on the l.h.s of the bounds a Gaussian distribution for the modulation in each bin.
- Then $p_{joint} = p_\eta \, p_A$ is the combined probability of obtaining the experimental result for that $\tilde{\eta}$. Then we maximize it w.r.t. $\tilde{\eta}$ to obtain the highest joint probability.

Other inelastic plots